

Precast Bridges: Design for Time Dependant Effects

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INTRODUCTION

Bridge construction with pre-fabricated elements, longitudinal beams and/or slab decks, is nowadays a common procedure all over the world. In this paper standard type of medium span bridges are analysed and, on one hand, a mathematical tool is proposed to take into account time effects and, on the other hand, the design considering both the prestress layout and the construction procedure is discussed.

At first, a mathematical formulation is presented that tries to improve the general and simplified formula to take into account creep effects, when the structural system is modified:

$$S_{\infty} = S_1 + \frac{\varphi}{1 + \chi\varphi} (S_2 - S_1)$$

Where stress resultants at long term, S_{∞} , are obtained as an intermediate value between the elastic value for the structural system at the time of construction, S_1 , and for the final structure, S_2 . The formulation developed in this paper has in mind the specific construction procedures adopted in bridge construction and tries to keep the basics and the simplicity of this expression.

The expressions show clearly the importance of the structural changes during construction and indicate the correct way to evaluate stress resultants, stresses and deformations considering a good evaluation for the creep coefficients. It is also pointed out as a design guidance, that the stress values at critical sections for structural systems 1 and 2, should be as close as possible, so that the structure is less sensitive to creep.

In the second part of this paper, based on a bridge designed and built recently in Portugal, the procedure developed is applied to study the influence of the prestress value and its layout on the structural behaviour characteristics and its economy. The advantages of a detailed analysis to obtain the more efficient solution is stressed. Another aspect that is discussed is the influence of the construction procedure on the value of the prestress needed. The efficiency of different construction methods is quantified for the case of road viaducts.

Keywords: concrete, prefabrication, creep effects, construction procedures

1. FORMULATION FOR TIME DEPENDANT ANALYSIS OF PREFABRICATED BRIDGES

In this sub-chapter a summarized mathematical deduction is presented for the formulation proposed on this paper that leads to the so called modified aging coefficient method. For this purpose some basic time dependant equations are remembered.

1.1 Basic Creep Concepts

Creep deformations are due to stresses applied along the time and are given by

$$\varepsilon_{c\sigma}(t, t_0) = \varepsilon_{ci}(t_0) + \varepsilon_{c\varphi}(t, t_0) \quad (1.1)$$

where

$$\varepsilon_{ci}(t_0) = \frac{\sigma_c(t_0)}{E_{c,t_0}} \quad (1.2)$$

is the initial deformation for a stress applied at a time t_0 , and

$$\varepsilon_{c\varphi}(t, t_0) = \varphi(t, t_0) \cdot \varepsilon_{c0}(t_0) = \varphi(t, t_0) \frac{\sigma_c(t_0)}{E_{c,28}} \quad (1.3)$$

is the deformation increase in time due to the same stress kept constant in time. So the final deformation may be expressed by,

$$\varepsilon_{c\sigma}(t, t_0) = \sigma_c(t_0) \left[\frac{1}{E_{c,t_0}} + \frac{\varphi(t, t_0)}{E_{c,28}} \right] \quad (1.4)$$

or, in an equivalent and more understandable way by

$$\varepsilon_{c\sigma}(t, t_0) = \frac{\sigma_c(t_0)}{E_c(t_0)} \cdot [1 + \varphi'(t, t_0)] \quad ; \quad \varphi'(t, t_0) = \varphi(t, t_0) \frac{E_{c,t_0}}{E_{c,28}} \quad (1.5 \text{ a and b})$$

or even,

$$\varepsilon_{c\sigma}(t, t_0) = \sigma_{c,t_0} J(t, t_0) \quad ; \quad J(t, t_0) = \frac{1 + \varphi'(t, t_0)}{E_{c,t_0}} \quad (1.6 \text{ a and b})$$

In the case of stress increments applied at different times, we have

$$\varepsilon_{c\sigma}(t, t_0) = \sigma_c(t_0) J(t, t_0) + \sum_{i=1}^n \Delta \sigma_{c,t_i} \cdot J(t, t_i) \quad (1.7)$$

The main difficulty with creep is its correct quantification. Numerous authors and code makers [1, 2, 4, 5, 9] have compiled experimental results and proposed different formulations but as Caldentey [2] has shown, the differences between experimental and theoretical results can still be very significant.

EC2 [5] pretends its formulation assures a mean variation coefficient of the order of 20% in comparison with laboratory results, but, as we know, there are still important variations when "in situ" measurements are considered.

1.2 Main Structural Effects

As is well known, if two structural elements subjected to different stress fields are put together, the free deformation due to creep is restricted, so compatibilization stresses will be generated (see Fig. 1.1). This is a typical situation for prefabricated construction where it is usual to connect elements with different age and/or stress histories. This compatibilization process implies imposing axial deformations and curvatures to the sections and, if the structure is hyperstatic, other stress resultants will develop.

Another usual situation with prefabrication is the introduction of significant modifications to the longitudinal structural system during construction, as illustrated in Fig. 1.2. In this case, creep structural deformations can't develop freely at the connections and, again, the need for deformations compatibilization is such that stress resultants redistributions will develop.

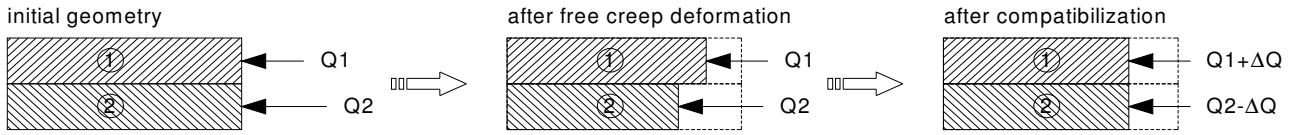


Fig. 1.1. Compatibilization of fibre deformations at a section level due to differential free creep

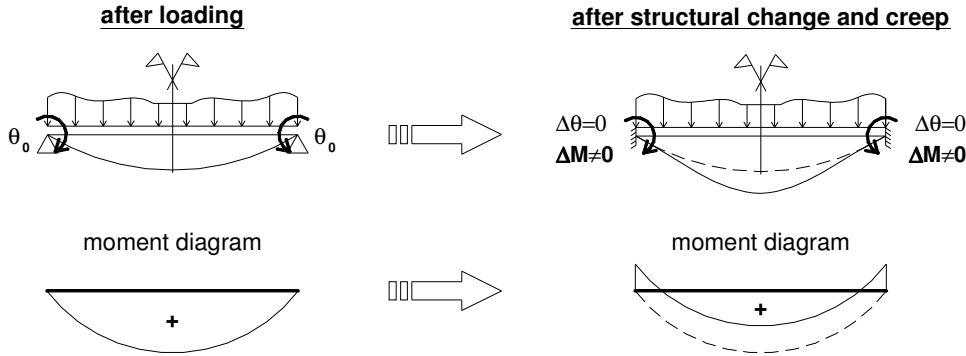


Fig. 1.2. Redistribution of moments in a structure due to rotation compatibilization at the beam ends

In designing prefabricated bridges these two structural effects are taken in the same analysis, eventually considering as well the effects of steel, that are not analysed here.

1.3 General Aging Coefficient Method

This method is based on the concept that creep deformation, due to a stress increment applied gradually in time, is smaller than would be in the case the total stress was maintained constant.

So, considering a gradual stress variation in time of $\Delta\sigma_c(t, t_0)$ and taking into consideration equations 1.6 and 1.7, we can write:

$$\varepsilon_{c\sigma}(t, t_0) = \sigma_c(t_0) \frac{1 + \varphi'(t, t_0)}{E_{c,t_0}} + \Delta\sigma_c(t, t_0) \frac{1 + \chi(t, t_0) \cdot \varphi'(t, t_0)}{E_{c,t_0}} \quad (1.8)$$

It is out of the scope of this paper the deduction of the χ value presented elsewhere [7, 9]. Its value, in the case of a stress variation equal to the relaxation function, is given by

$$\chi(t, t_0) = \frac{1}{1 - \frac{R(t, t_0)}{E_{c,t_0}}} - \frac{1}{\varphi'(t, t_0)} \quad (1.9)$$

Expression (1.8) is usually presented as

$$\varepsilon_{c\sigma}(t, t_0) = \frac{\sigma_c(t_0)}{E_{c,eq}} + \frac{\Delta\sigma_c(t, t_0)}{E_{c,aj}} \quad (1.10)$$

$$\text{where } E_{c,eq} = \frac{E_{c,t_0}}{1 + \varphi'(t, t_0)} \quad (1.11)$$

represents the equivalent elasticity modulus

$$\text{and } E_{c,aj} = \frac{E_{c,t_0}}{1 + \chi(t, t_0) \varphi'(t, t_0)} \quad (1.12)$$

is the adjusted elasticity modulus used to analyse the concrete behaviour under a gradual stress variation. In the following paragraph these general time dependant concrete behaviour characteristics are applied to

the usual construction procedures for prefabricated bridges, in order to obtain a more general, fiable and simple expressions, to evaluate the structural response.

1.4 Modified Aging Coefficient Method

The formulation developed in this paragraph and proposed for the design of prefabricated concrete bridges is based on the general aging coefficient method pointed out by other authors [3, 6, 10], but with the introduction of some simplifications for this situation, in particular the division of time in 3 instances:

- t_0 – time of prefabrication of the beams considering the action of its selfweight and prestress.
- t_1 – time of construction and the begining of service life. It is considered that, due to a relative short construction time, the creep effects are not significantly affected by this approximation.
- t_∞ – time at the end of the bridge service life.

Concerning the structural systems, three types of structures can be considered:

- Initial structural system, usually the isostatic prefabricated elements, that represents the structure for the period $[t_0, t_1]$
- Intermediate structural systems that represent the structure after certain modifications during construction that can be at a section level or by establishing continuity between beams. For a certain construction procedure different intermediate systems may be considered for each action, depending on the situation at the time they are applied. For example, to analyse the total or partial load due to the deck, depending on the situation, it could be the case to adopt the isostatic system mentioned above or this one after establishing beam continuity. In fact, as the superposition principle is applicable, creep effects can be analysed separately for each action but limited, in this proposal, to the three time periods refered to before.
- Final structural system that corresponds to the structure at the beginning of its service life and afterwards.

It should be mentioned that in the following procedure the initial and intermediate structural systems are considered as system 1 and the final system is always taken as system 2.

The procedure adopted is based firstly on a classical section analysis by the force method, based on four steps, described below.

Step 1: load application to the initial structure

Step 2: restriction of the main beam creep deformation that would occur between t_1 and t_∞ , if no structure modification occurred: $\Delta\varphi'_1 = \varphi'_1(t_\infty, t_0) - \varphi'_1(t_1, t_0)$ (1.13)

Step 3: establishment of external equilibrium

Step 4: sum of the previous steps (long term response)

The results of the stresses expressions are presented below at table 1.1, for the main beam (primary element) and as well for the complementary concrete slab and, in the following, for curvatures and deformations. The full deduction is presented in a MSc. thesis [8].

	Main beam stresses	Complementary slab stresses
Step 1	$\sigma_{1,i}^P = \frac{N}{A_1} + \frac{M}{W_{1,i}}$	$\sigma_{1,i}^C = 0$
Step 2	$\sigma_{2,i}^P = -\frac{\Delta\varphi'_1}{[1 + (\chi\varphi)_1]_{ef}} \left(\frac{N}{A_1} + \frac{M}{W_{1,i}} \right)$	$\sigma_{2,i}^C = 0$
Step 3	$\sigma_{3,i}^P = \frac{\Delta\varphi'_1}{[1 + (\chi\varphi)_1]_{ef}} \left(\frac{N}{A_{2\text{hom}}} + \frac{M_2}{W_{2\text{hom},i}} \right)$	$\sigma_{3,i}^C = \frac{\Delta\varphi'_1}{[1 + (\chi\varphi)_1]_{ef}} \left(\frac{N}{A_{2\text{hom}}} + \frac{M_2}{W_{2\text{hom},i}} \right) k_{\text{hom}}$
Step 4 (long term)	$\sigma_{\infty,i}^P = \overline{\sigma}_{1,i}^P + \frac{\Delta\varphi'_1}{[1 + (\chi\varphi)_1]_{ef}} \left(\overline{\sigma}_{2,i}^P - \overline{\sigma}_{1,i}^P \right)$	$\sigma_{\infty,i}^C = \overline{\sigma}_{1,i}^C + \frac{\Delta\varphi'_1}{[1 + (\chi\varphi)_1]_{ef}} \left(\overline{\sigma}_{2,i}^C - \overline{\sigma}_{1,i}^C \right)$

Table 1.1. Stress calculation steps and final stress expressions

In the previous expressions:

$[1+(\chi\varphi)_1]_{ef}$ replaces $(1+\chi\varphi)$ in the classical equation presented in the introduction, and is given by the following equation, where all the coefficients refer to the pre-fabricated beam concrete.

$$[1+(\chi\varphi)_1]_{ef} = [1+\chi_1(\infty, t_1) \cdot \varphi'_1(\infty, t_1)] \cdot \frac{E_{1,t0}}{E_{1,t1}}$$

$\sigma_{n,i}^x$ represents the stress increments in the main beam (X=P) or complementary slab (X=C) for the steps $n = 1, 2$ and 3 of the calculation procedure at fibre i

$\overline{\sigma_{n,i}^x}$ represents the stresses in the main beam (X=P) or complementary slab (X=C) acting on the initial structure ($n=1$) or final structure ($n=2$) at fibre i

k_{hom} represents the homogeneity coefficient given by the relation between the adjusted elastic modulus of the complementary and primary elements, referred to age t_1

$$k_{hom} = \frac{E_{c,t1,aj}}{E_{1,t1,aj}} = \frac{E_{c,t1}}{E_{1,t1}} \times \frac{1+\chi_1(\infty, t_1) \cdot \varphi'_1(\infty, t_1)}{1+\chi_c(\infty, t_1) \cdot \varphi'_c(\infty, t_1)}$$

For area (A) and flexion modulus (W) the lower symbols 1 and 2hom stand for initial section and final homogeneized section, respectively. It should be noticed that in the case of step 3 the moment M_2 is referred to the center of gravity of the homogeneized section.

The presented long term stress formulas (step 4) are, on one hand, similar to the basic one referred at the introduction but clarifies the meanings of the variables and, on the other hand, within the simplification introduced by the general aging coefficient method and by the time division adopted, give results as correct as obtained on an analysis step by step.

Concerning curvatures and deformation, the force method leads to the following formula:

$$\left(\frac{1}{R}\right)_{\infty} = (1+\varphi'_1(t_1, t_0)) \cdot \overline{\left(\frac{1}{R}\right)_1} + \Delta\varphi'_1 \cdot \overline{\left(\frac{1}{R}\right)_2}$$

$$\delta_{\infty} = (1+\varphi'_1(t_1, t_0)) \cdot \overline{\delta_1} + \Delta\varphi'_1 \cdot \overline{\delta_2}$$

Where $\overline{\left(\frac{1}{R}\right)_n}$ and $\overline{\delta_n}$ are the curvatures and the deformations obtained in the initial structure ($n=1$) with

flexural stiffness $E_{1,t0} I_1$ or in the final structure ($n=2$) with flexural stiffness $E_{1,t0} I_{2hom}$.

In the analysis of structural hyperstaticity it is shown in [8] that these expressions keep its validity.

2. APPLICATIONS TO PREFABRICATED BRIDGES

Based on the geometry and construction procedure adopted in a recently built viaduct, and after the analysis of its behaviour in time, the prestress quantity and its layout is discussed in order to improve its efficiency in terms of behaviour characteristics and economy. The influence of the quantity and cable layout with different construction procedures is also analysed in this chapter.

2.1 Analysis of a Case Study

In Fig. 2.1 the longitudinal and transverse geometry of the viaduct is presented and Fig. 2.2 shows the details of the support and prestress steel used to establish the connection between the prefabricated girders.

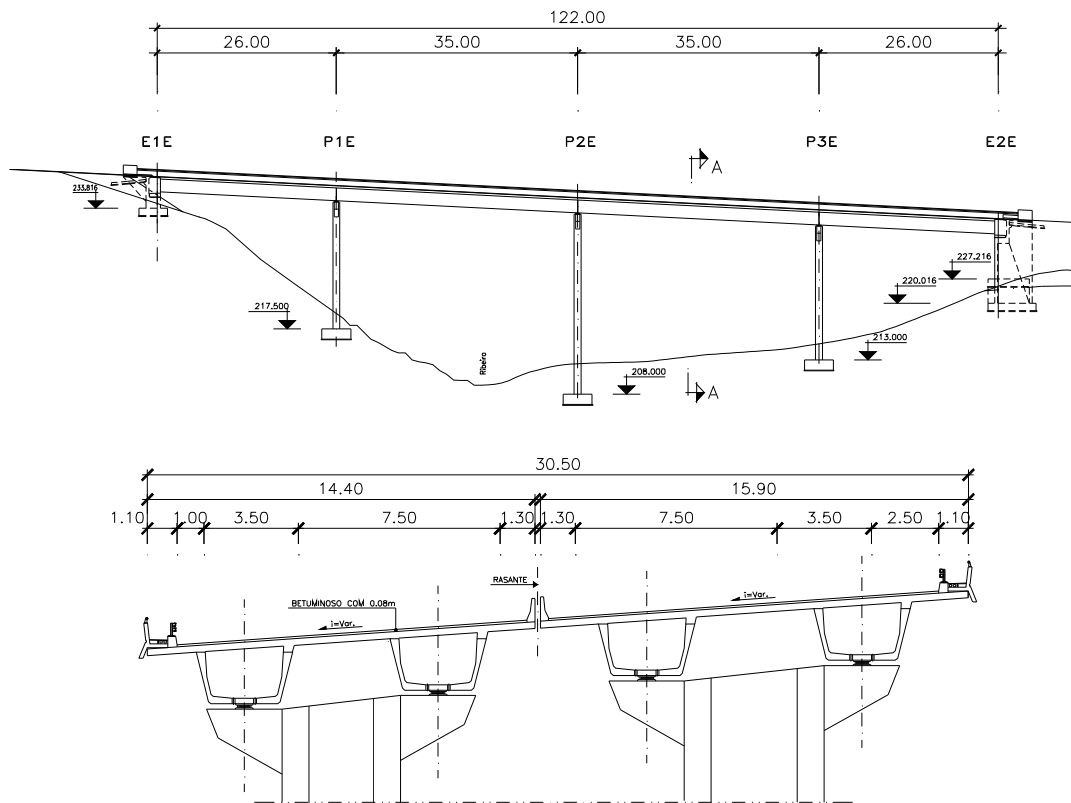


Fig. 2.1 Longitudinal and transversal geometry of the bridge considered for the case study

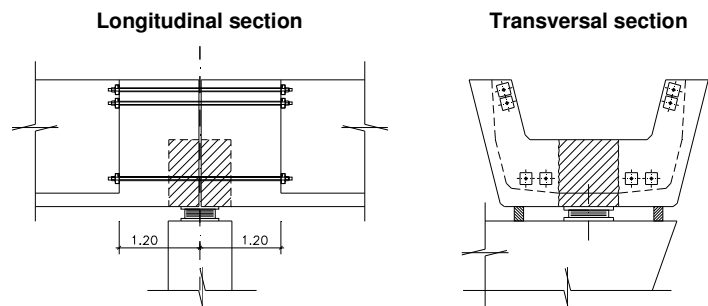


Fig. 2.2 Details of the connection between prefabricated beams

In order to simplify the analysis only half the section was considered and the transversal inclination was eliminated as shown in Fig. 2.3. The main section fibres for the stress evaluations are also presented. The variable actions were valued in order to give an equivalent section effects as if the total bridge was considered.

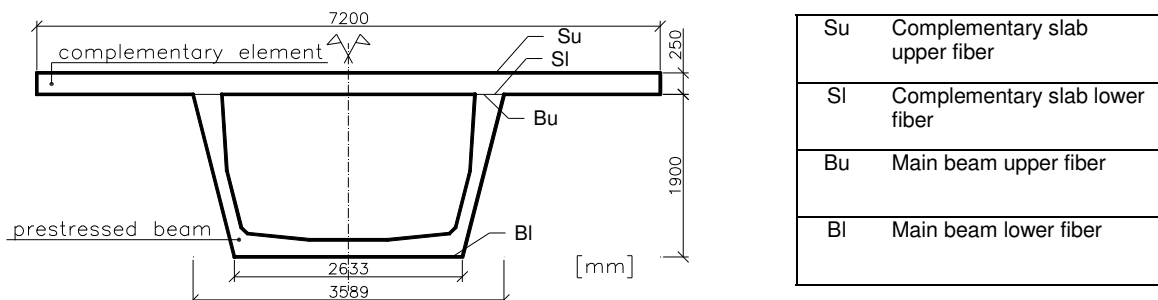


Fig. 2.3 Section geometry considered in the analysis and main fibres identification

The bridge was designed with no continuity prestress over the supports and the construction procedure were as follows – see Fig. 2.4:

- Phase 1 – Positioning of the prefabricated beams
- Phase 2 – Execution of the beams connection as presented in Fig. 2.2 giving continuity to the system.
- Phase 3 – Concreting of the top slab for 6.0m to each side of the supports
- Phase 4 – Execution of the remaining slab deck
- Phase 5 – Finalising the bridge deck (non structural elements).

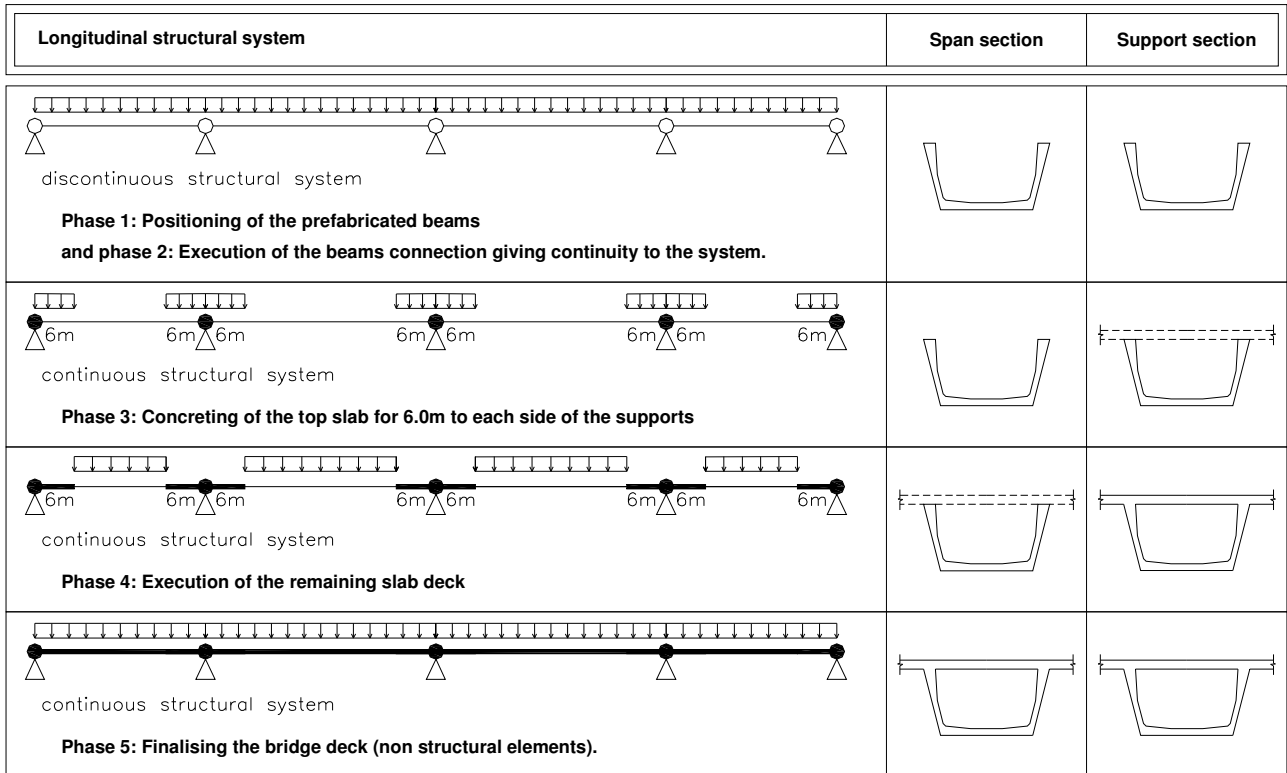


Fig. 2.4 Construction phases adopted in the case study

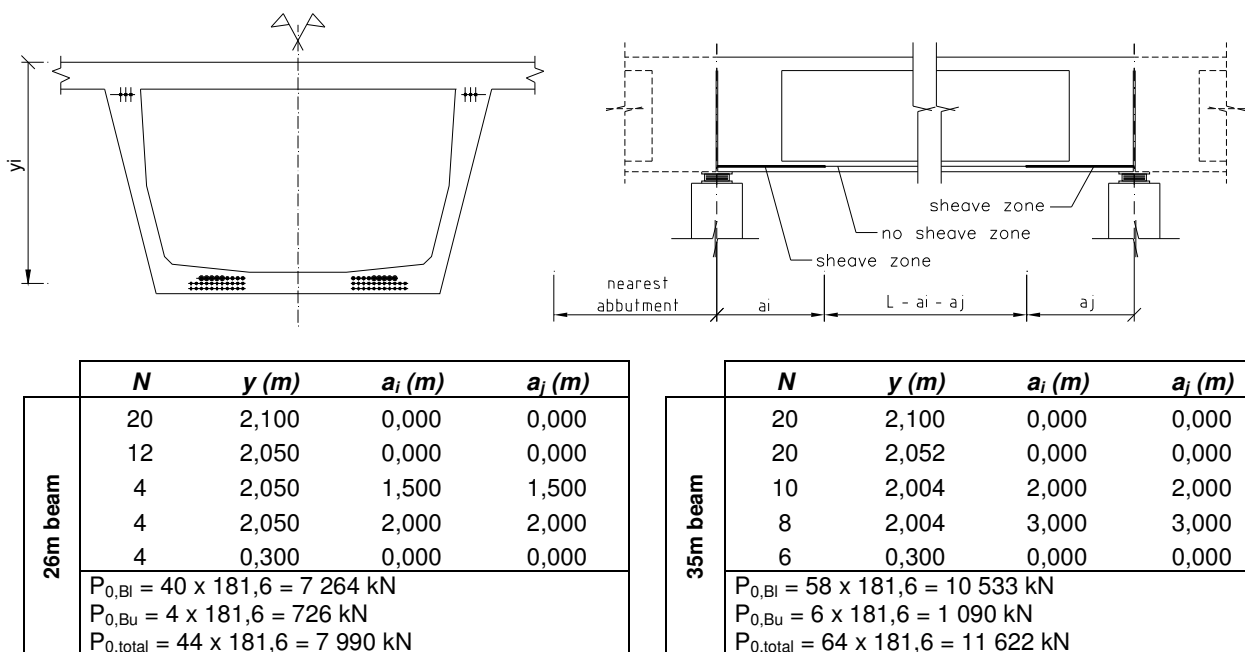


Fig. 2.5 Prestressed layout adopted in the design

The quantities of prestress and its layout, adopted in the design, are summarized in Fig. 2.5, where the strand lengths with plastic sheaves are also shown. It should be noticed that the strand lengths with no adherence to concrete are in this case short. In fact, 82% and 72% of the strands, for the lateral and central spans respectively, are kept with no sheaves and these are taken at the most till 20% of the span length. As a simplification for the analysis it was considered that the effects of each strand was totally effective after a transfer length of 1.5m with a linear variation within this length.

The time dependant material characteristics considered in the analysis followed MC90, considering a relative humidity of 80%, and are summarized in table 2.1.

Concrete class	Main beam	Slab
	C45/55 (rapid hardening cement)	C30/37 (normal hardening cement)
$E_{c,t0}$ (GPa)	30,5	N/A
$E_{c,t1}$ (GPa)	36,0	26,1
E_{cm} (GPa)	36,0	32,0
$\phi(t_1; t_0) / \phi'(t_1; t_0)$	0,74 / 0,63	N/A
$\phi(t_{\infty}; t_0) / \phi'(t_{\infty}; t_0)$	1,99 / 1,69	N/A
$\phi(t_{\infty}; t_1) / \phi'(t_{\infty}; t_1)$	1,45 / 1,45	2,84 / 2,31
$\chi(t_{\infty}; t_1)$	0,83	0,59

Table 2.1 Time dependant material characteristics considered in the analysis

The analysis followed the methodology described in paragraph 1.4 considering three timings, t_0 , t_1 and t_{∞} , respectively, for the pre-fabricated beams execution, time of bridge construction and at long term.

The stress distribution along the viaduct for the bottom fibre of the pre-fabricated beams is shown in Fig. 2.6, for the construction phases and during the service life. It can be seen that during construction compression is guaranteed easily, and, during the service life, the limit state of decompression is verified in the middle span but with an unnecessary compression level on the rest of the span.

The moment distributions during the service life are presented in Fig. 2.7 for time t_1 and t_{∞} , considering, for each one, the variation due to the variable loads positioning. It can be noticed, that there is an important redistribution of moments with time from the supports to the span, due to the fact that the influence of prestress is more important than that of the loads. However there is never an inversion of the moment sign, at least for the quasi-permanent loads, sometimes considered as an inconvenient of this type of construction. A question that could be raised, and will be discussed further on, is if it wouldn't had been possible to decrease the prestress value, by diminishing the unfavourable effect of the hiperstatic moment. This could be done by increasing the cable sheaves lengths.

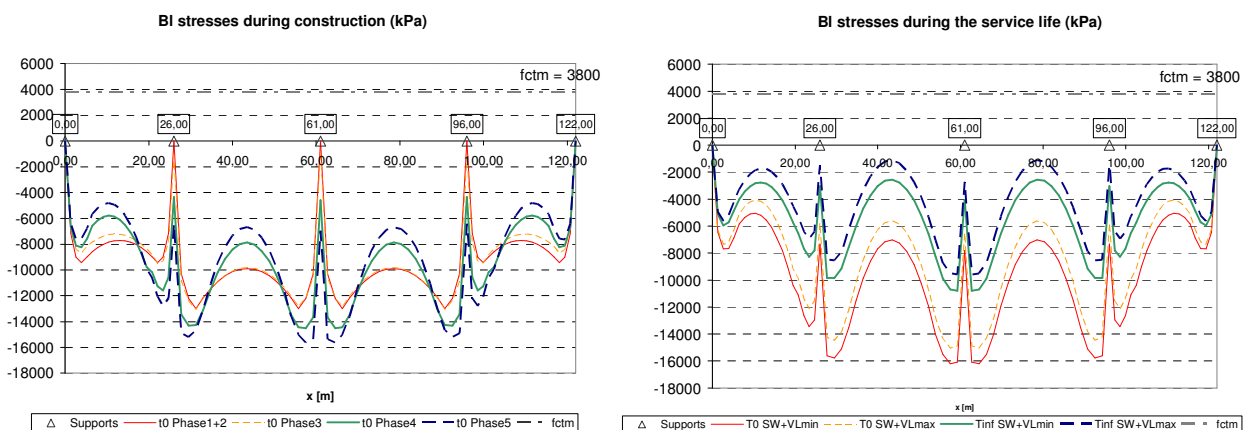


Fig. 2.6. Stress distribution during construction and for the quasi-permanent loads at the bottom fibre of the prestressed beam

For all the main section fibres the stress envelopes are presented in Fig. 2.8, for the slab (upper and bottom fibres) and for the upper fibre of the prefabricated beam and it can be observed that tension is limited to 3 MPa and 4 MPa, respectively, showing that although cracking is possible, crack opening can easily be controlled. These results show, in our opinion, a reasonable good design criteria.

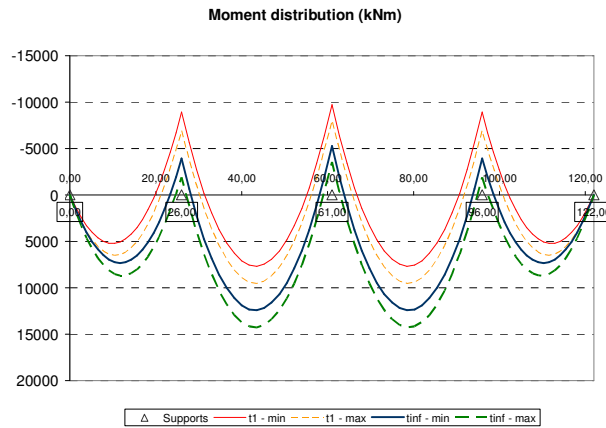


Fig. 2.7. Moment distribution during the service life of the bridge for the quasi-permanent loads

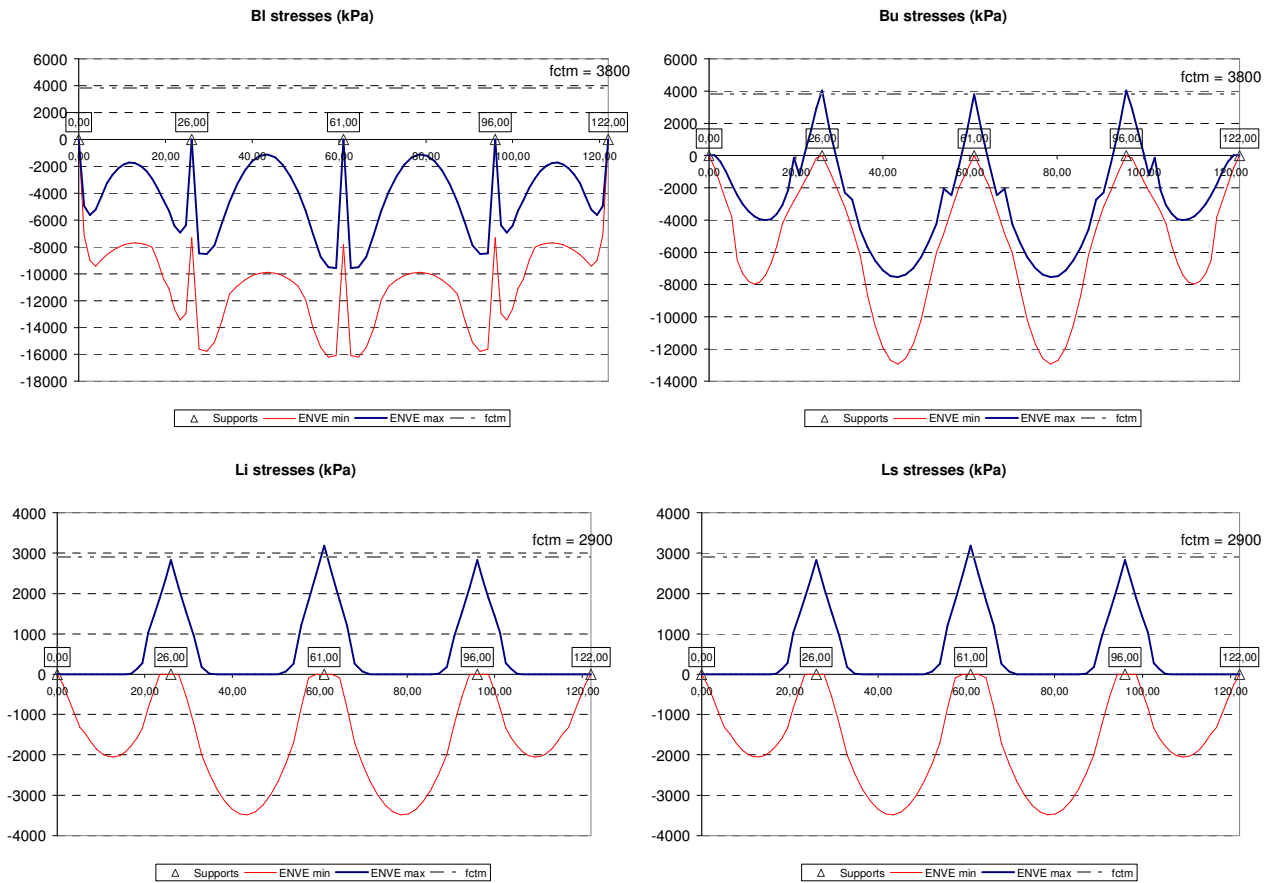


Fig. 2.8 Stress limits during the service life for all section critical fibres

The phasing of the deck slab, with a previous execution of the slab near the supports (phases 3 and 4), has been efficient limiting the tension at the top beam fibre, as can be seen by the discontinuity at the stress diagram of the main beam upper fibre.

2.2 Cable Layout Modification

Taking into consideration the same structure and construction procedure an analysis was performed, for a different cable layout and prestress value, in order to verify if a better structural efficiency and economy was possible. With this purpose an alternative design was tested in such a way that should guarantee the same compression level at the bottom fibre (1 MPa). A solution with less 10% of cables and bigger sheaves

lengths was designed with a prestress force variation along the span as shown in Fig. 2.9, where it is compared with the previous base solution.

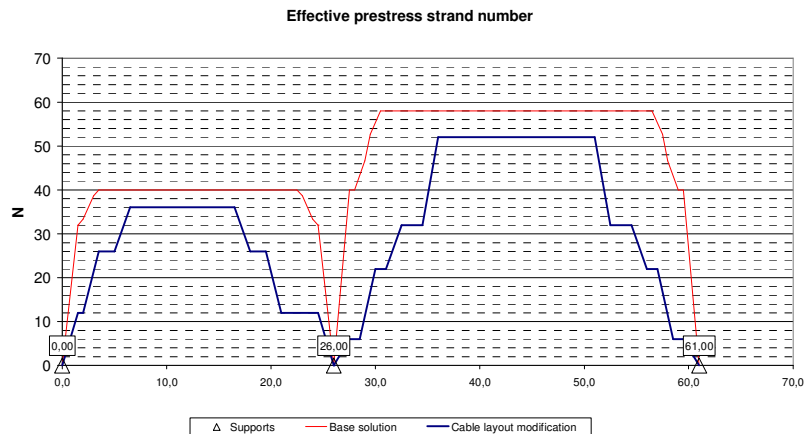


Fig. 2.9 Prestressed layout adopted in the design

It is interesting to see that the stress distribution during the structure service life presented in Fig. 2.10 insures the objective defined above and, at the same time, gives a much smaller stress variations with time. This makes the structure much less sensible to variations of the time dependant parameters. This was possible by a significant reduction of the hiperstatic prestress moment due to two effects: more efficient lay-out and smaller prestress. It can be seen at Fig. 2.10 that the reduction of the hyperstatic moments made the stress diagrams, at the initial and final time (t_1 and t_∞), get closer.

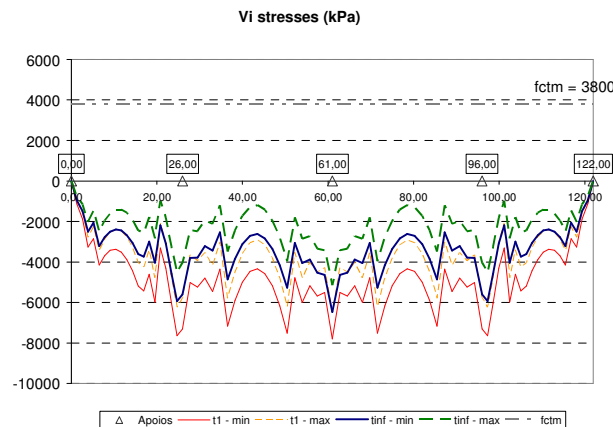


Fig. 2.10 Stress distribution for the quasi-permanent loads at the bottom fibre of the prestressed beam

2.3 Construction Procedure

To analyse the influence construction procedure has in the design and behaviour of this case study, two different procedures were analysed:

- C: Positioning of pre-slabs 0.1m thick, on all the bridge length over the simple-supported pre-fabricated beams before connecting them and keeping, otherwise, the construction procedure.
- D: Execution of all the slab deck continuously over the simple supported prefabricated beams.

In order to obtain efficient solutions the prestress quantities and the layout adopted are presented in Fig. 2.11, in comparison to the previous cases. It should be mentioned that for the first case (C) the prestress level had to be somehow bigger, but for the second situation, case D, there was a need for a significant prestress force increase. Concerning the prestress layout it can be seen that the cables sheaves lengths were decreased so that the prestress diagram could be closer to the moments diagrams due to the quasi-permanent loads (more like a simple supported distribution). This situation is more obvious for the denominated case D.

Concerning the stress distribution during the service life, for these two cases, the results obtained showed that it was possible to obtain stresses with the same order of magnitude and type of distribution along the span.

However comparing the moment distributions of case A with the one obtained for case D (continuity only for non-structural elements and time effects), it can be seen in Fig. 2.12 that the moments over the support are clearly smaller for case D due to the fact that the deck weight was applied over the simple supported system.

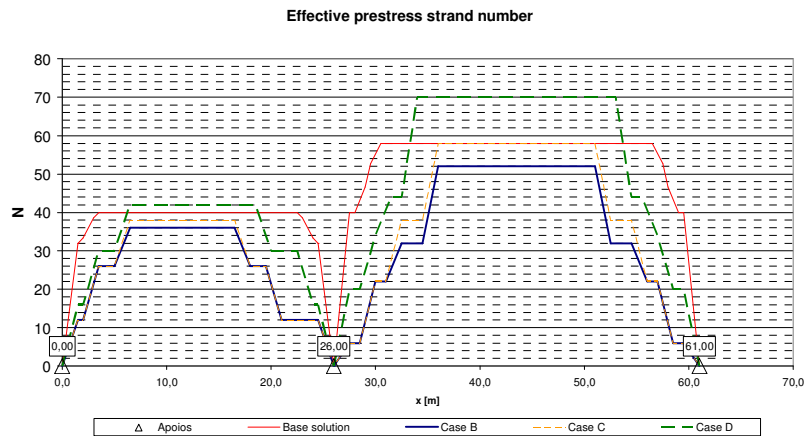


Fig. 2.11 Prestressed layout adopted in the design

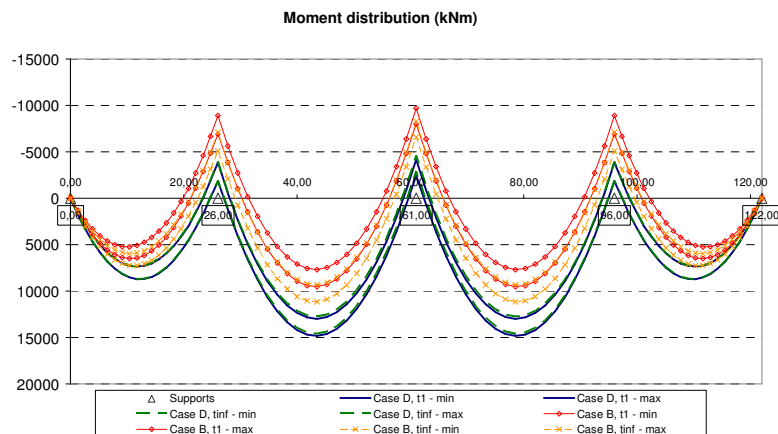


Fig. 2.12 Comparison of the moment distribution of cases B and D during service life

As summarized in table 2.2, the values obtained for prestress at long term, keeping the same design criteria (stress limits), had significant differences when the cable layout or the construction procedures were changed.

	P [kN]	P/P _B
A	8950	111.5%
B	8030	100.0%
C	8950	111.5%
D	10800	134.5%

Table 2.2 Prestress after time dependant losses in the different cases analysed

It was clear, from this study, that to obtain a less sensible structure to time effects and at the same time obtain some prestress economy, the cable layout should be carefully defined. The analysis showed as well that the construction procedure influences the prestress value and its layout in order to obtain equivalent structural performances. For the order of magnitude of the relation quasi-permanent loads to selfweight of road viaducts the study showed that the construction procedure adopted in the case study is very efficient. However for other design values of that relation, as for example train bridges, for normal or high speed, it is natural that construction procedures more similar to case C, or even case D, could be more efficient.

3. CONCLUSIONS

A design procedure to analyse the time dependant effects due to the modification of the structural system in the construction of prefabricated standard bridges is presented. The formulation shows in a very rational way how the stress resultants, stresses and deformations can be obtained through a correct estimation of the creep coefficients or its increments and of the elastic values obtained from the structural systems at the time the action is applied and at the end of construction.

The formulation was applied to a real viaduct project and showed its efficiency. In this paper it was shown how a careful layout of the prestress strands, particularly its sheave lengths, can grant better structural behaviour and economy.

Other construction procedures were as well analysed with this formulation showing the need for different prestress values. For road viaducts it was clear, from the results, that beam continuity during construction is favourable.

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