

STRESS FIELD MODELS FOR STRUCTURAL CONCRETE

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SUMMARY

Strut and tie models (*STM*) are currently recognized as a valuable tool for the design and detailing of discontinuity regions in structural concrete. These regions are still weakly covered in technical documents, although its deficient design and detailing is frequently related to damages and even failures. Uniqueness and models validity is frequently mentioned as an obstacle for its practical implementation. A new approach, based on energetic criteria, applied to stress field models, is proposed to perform the assessment of the models. The method was tested by comparison with experimental results and known solutions for some typical cases. Its application to practical design situations is illustrated as well. The development and the practical application of such specific design tools will improve consistency and clearness of design methods, contributing to *keep concrete attractive*.

1. INTRODUCTION

The application of strut and tie models (*STM*) will improve consistency in design concepts, allowing the treatment of discontinuity and current regions with comparable accuracy and emphasizing the essential role of detailing in the design process. Based on equilibrium and plasticity, such models intend to reproduce the stress trajectories, showing the flow of forces, throughout a concrete region. In a simplified way, service behaviour can be indirectly checked by ensuring that the model follows the elastic stress fields. In spite of such simple concepts, for different reasons the practical application of the method can not be considered widely disseminated. Some of the main aspects frequently mentioned refer to the model assessment validity and to its uniqueness.

2. MODEL UNIQUENESS

The establishment of an equilibrate strut and tie model requires some engineering judgement. Modelling principles were printed out, e.g., by Schlaich et al (1987, 1991) and later by Schafer (1996). The designer would easily achieve the corbel design model shown at fig. 1, although, some doubts could come forward if the same corbel suspends the load. In that case, it is possible to transmit the load to the supports by vertical or inclined reinforcement, thus two different *STM* are possible (see fig. 2). Based on the lower bound of the plasticity theory, both models are correct, if the structure has enough ductility to adapt itself to the chosen

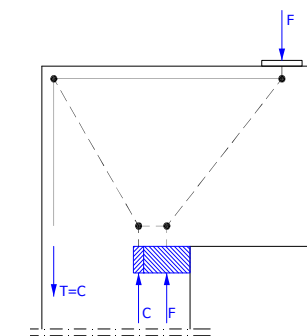


Fig. 1 - Simple STM for a corbel

model. A question remains: Which part of the load F should go through inclined stirrups, in order to avoid deficient service behaviour and assure enough ultimate resistance? In most cases the answer is based on the elastic theory or specific tests. Schlaich et al (1987) mentioned that the main struts and ties should follow the elastic stress trajectories in order to indirectly check serviceability. This criterion often leads to hyperstatic models obtained by superposition of two different models (see fig. 3).

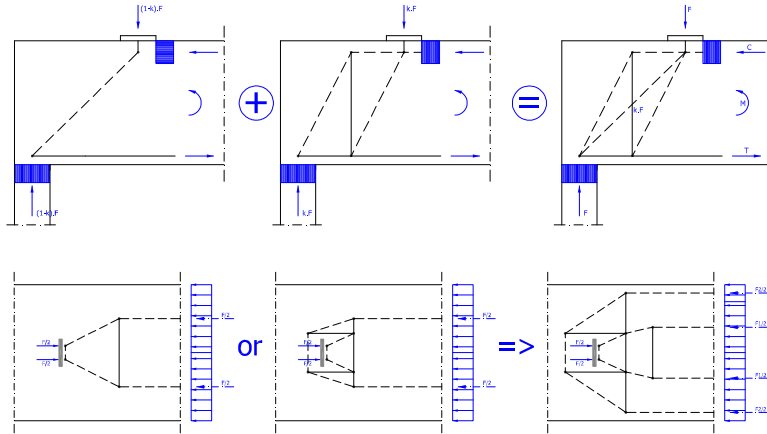


Fig. 3 – Hyperstatic models: a) concentrated load near support; b) internal anchorage load.

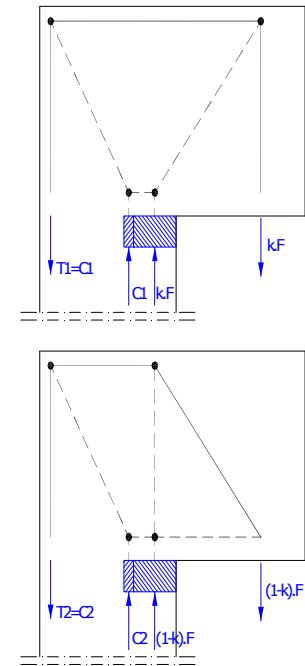


Fig. 2 – STM for a corbel with indirect load.

3. STRESS FIELD MODELS

The *STM* refinement can be the basis for the stress field model (*SFM*) which becomes very useful for many design issues, namely: -nodes geometry definition; -identification of distributed or concentrated ties; -definition of mechanical properties for the struts; and many others. For the illustration of *SFM* application a practical example of a viaduct fixed abutment shear wall is chosen, subject to several loads: -soil pressure; -deck's vertical loads; -horizontal loads. Fig. 4 represents the shear wall loads and corresponding *STM*.

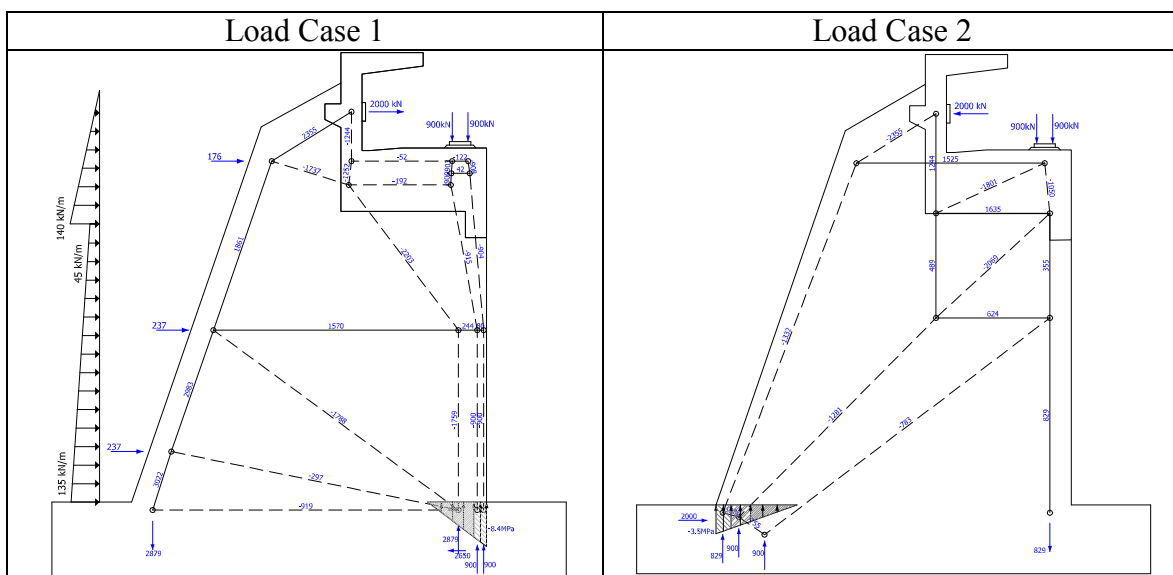


Fig. 4 – Abutment shear wall, loads and strut and tie model.

The *STM* gives a good approximation of the *D'region* structural behaviour, in general better than other current design techniques, and provides good guidance for detailing. As known, the struts are defined by a single line that represents compression stress fields on concrete. The check of stresses along struts can be disregarded, because it spreads inside the region, and only concentrated nodes should be checked (see fig.5). A step forward could be the development of the *SFM* illustrated at fig. 6, giving an excellent overview of the stress flow throughout the region.

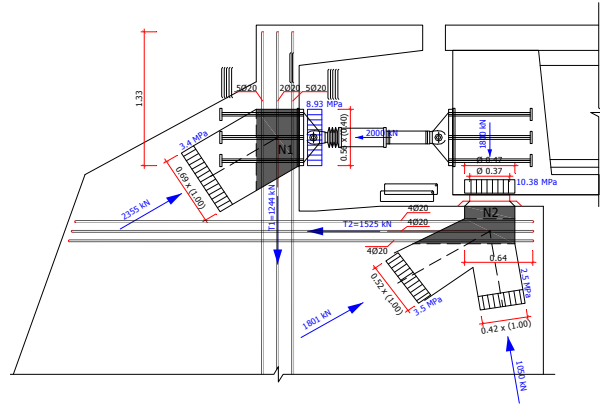


Fig. 5 – Check of nodes

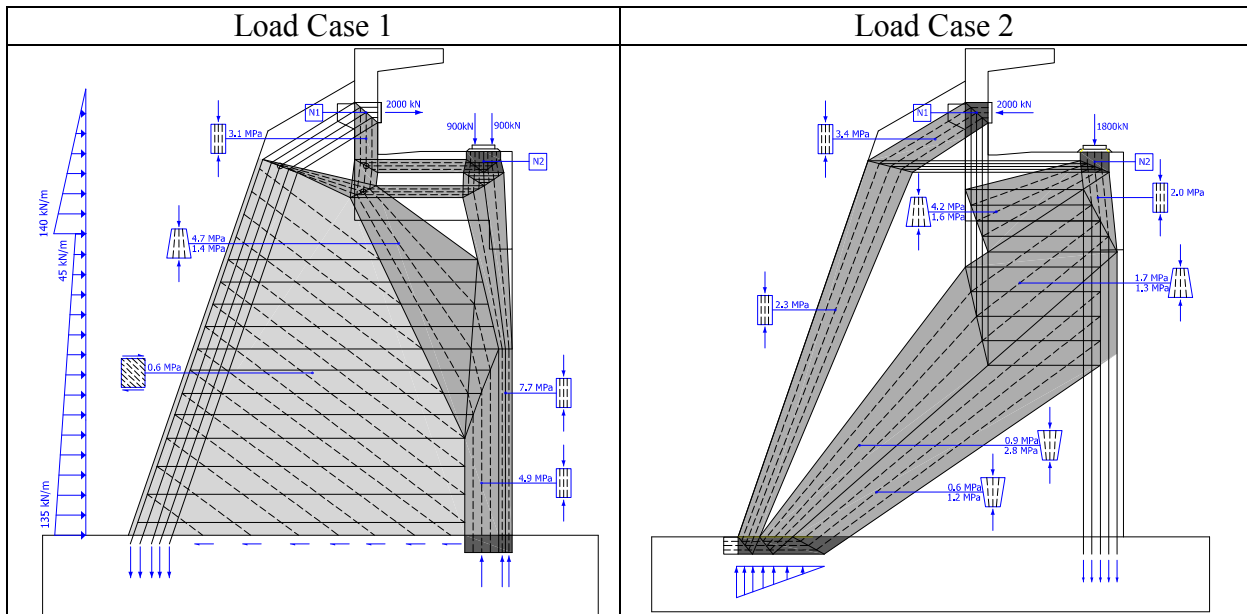


Fig. 6 – Abutment shear wall stress field model

4. MODELS ASSESSMENT

Following the stress fields definition associated to a *STM*, struts and ties mechanical properties evaluation are possible. Energetic techniques can be applied in order to consider compatibility in the general model allowing the development of assessment techniques. For every possible static condition, the “exact” solution minimizes the functional (Koiter, 1960).

$$\Pi = \frac{1}{2} \int_V \sigma^T \varepsilon dV - \int_{\Gamma} t^T u_{\Gamma} d\Gamma_u$$

σ – stress; ε – deformation; t – boundary loads; u_{Γ} – imposed displacements; Γ_u – cinematic boundary

Applying energetic theorems associated to *SFM*, it is possible to perform a nonlinear analysis, assuming nonlinear behaviour for concrete and tension stiffening effects (Sundermann, 1994). The proposed methodology can as well give some guidance for practical design and is based on deformation energy minimization, $dU/dF=0$, where F represents *STM* element forces.

Considering elastic behaviour for concrete, the energy of each strut and each tie can be calculated as indicate at fig. 7:

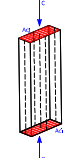
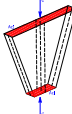
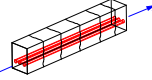
	$U = \frac{1}{2} \frac{C^2 L}{E_c A_c}$	<p>C, T – compression and tension forces, respectively A_c – prismatic strut area. A_{ci}, A_{cj} – initial and end areas E_c, E_s – Young modulus of concrete and steel L – strut or tie length ϵ_{sm} – reinforcement mean strain</p>
	$U = \frac{1}{2} \frac{C^2 L}{E_c (A_{cj} - A_{ci})} \cdot \ln\left(\frac{A_{cj}}{A_{ci}}\right)$	
	$U = \frac{1}{2} \frac{T^2 L}{E_s A_s} \text{ or generally } U = \frac{1}{2} T L \epsilon_{sm}$	

Fig. 7 – Struts and ties deformation energy

As an illustration, results were compared with deep beam tests performed by Leonhardt and Walther (1966) (see fig. 8). Large redistribution of internal stresses due to cracking was observed. In fact, *WT2* test ultimate load was approximately the double of the obtained by elastic based *STM*. The inner lever arm *z* tends to use the deep beam full height, thus reducing considerably the bottom tie force. In case of the deep beam *WT3*, the bottom reinforcement area was doubled, stiffening the bottom tie. The corresponding inner level arm *z* at failure is smaller than that of deep beam *WT2*.

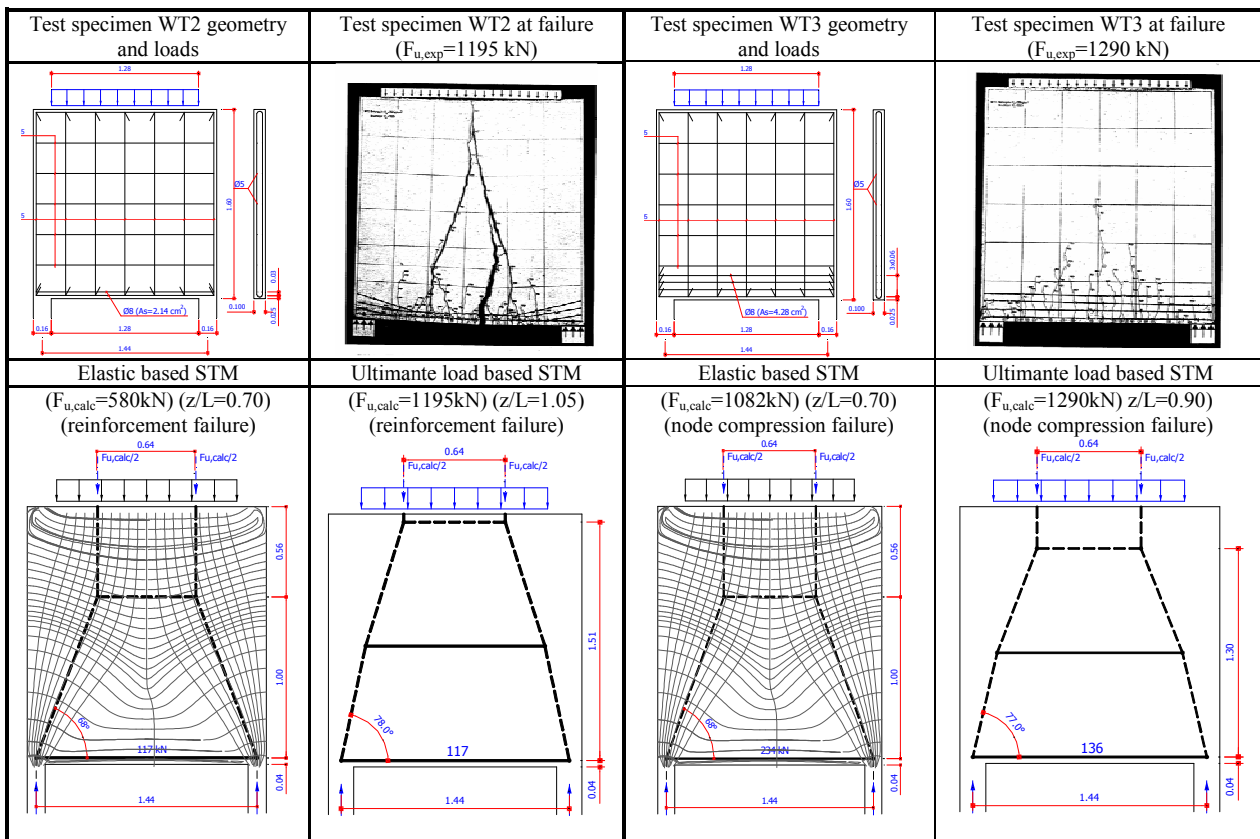


Fig. 8 – Deep beam tests (Leonhardt and Walther, Stuttgart, 1966)

Fig. 9 shows the numerical results obtained by minimizing the *SFM* total deformation energy, pointing out the different z/L values for each analysis: -elastic based *SFM* and ultimate load *SFM* for deep beam's *WT2* and *WT3*. This technique provides the designer a simple tool for the prediction of structural behaviour of *D*'regions and for the *STM* assessment validity.

5. HYPERSTATIC MODELS

The *STM* for interior tendon anchorage is a hyperstatic model, since part of load must be tied back behind the anchorage plate. The *SFM* for this case is shown at fig.10 as well as the variation of deformation energy with the percentage of load tied back. The minimum energy is obtained for $F_2 \approx 0.20F$, which is in accordance with *fib* recommendations 1999.

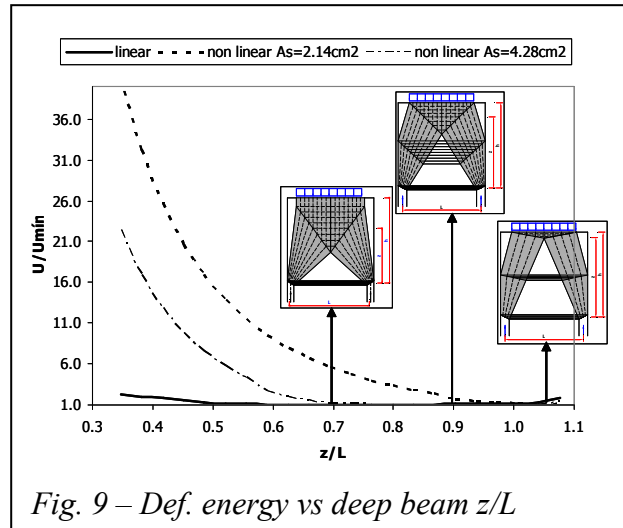


Fig. 9 – Def. energy vs deep beam z/L

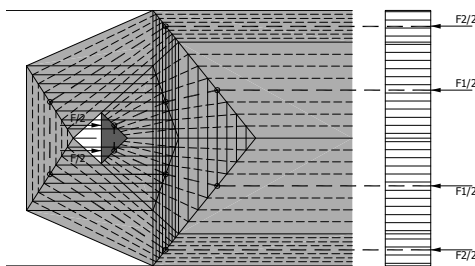
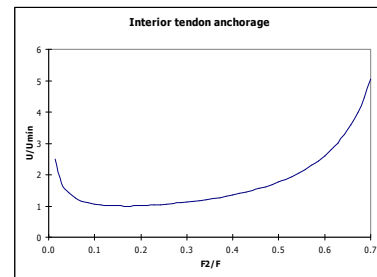


Fig. 10 – *SFM* for an interior tendon anchorage.



STM for a concentrated load near support should predict that part of the load that must be carried by the stirrups, as Schlaich (1991) has shown with corbel tests. MC90 and *fib* recommendations 1999, provides a simple equation for the suspended load percentage as a function of a/z . Fig. 11 compares the elastic solution (plate elements, finite element analysis), the MC90/*fib* recommendations 1999 and the proposed method numerical results.

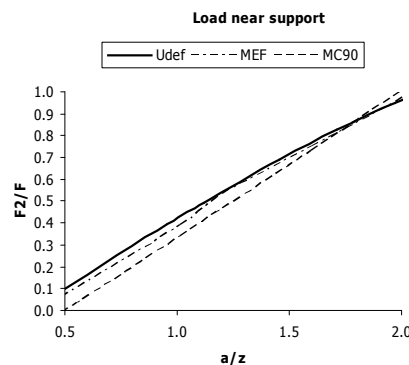
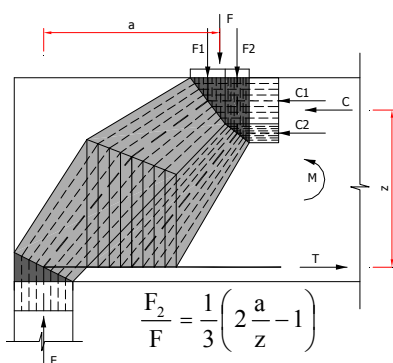


Fig. 11 – Load near support

The presented results illustrate the application of the proposed technique to the analysis of hyperstatic models.

6. PRACTICAL APPLICATIONS

The technique proposed has been applied to some practical design cases. Fig. 12 shows a cross section of a 750m long viaduct in Lisbon, with current spans of 52.3m and two major spans of 90m. It is an urban viaduct, near the ground, so aesthetics aspects were carefully considered. Fig. 13 shows the column *STM* and *SFM* for vertical loads. The *STM* “outer part” is quite similar to a reverse deep beam with a distributed load at the top. Due to the column top geometry and the load eccentricity, it is necessary to consider an *U*'loop represented by the model “inner part”. The inner level arm for the “deep beam part” is well known and was considered approximately 0.7 of the span. Nevertheless some doubts occur for the *U*'loop inner level arm z , which is most relevant for the horizontal tie force and for the vertical reinforcement anchorage. Minimizing deformation energy (fig. 13), a value of $z=3.2\text{m}$ was obtained and used for the columns design and detailing.

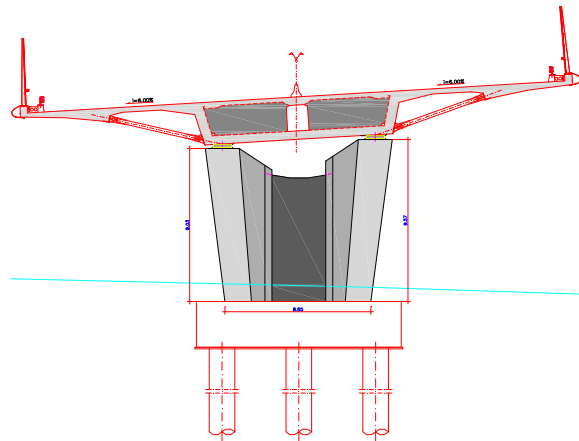


Fig. 12 – Cross section

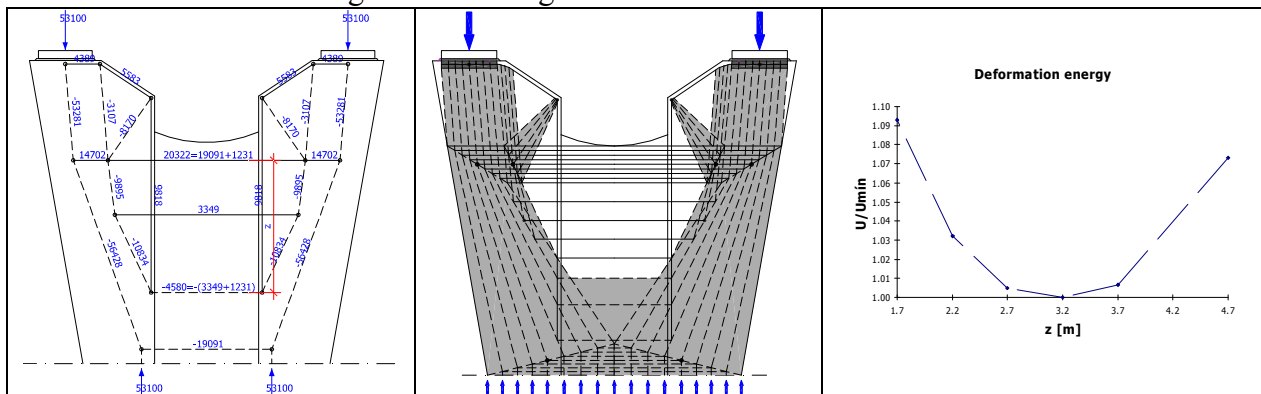


Fig. 13 – Vertical loads *STM* and *SFM* and respective deformation energy function of z .

7. FINAL REMARKS

The paper points out the importance of the development and application of stress field based models to the design and detailing of structural concrete discontinuity regions. The obtained results and its comparison with experimental results and the proposed solutions to hyperstatic standard cases, show the adequacy of the presented methodology to deal with uniqueness and assessment/validity “problems”, frequently referred in *STM* design field.

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